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ABSTRACT

This monograph concerns the geometry curriculum as it should be in an effective and complete mathematics program in grades K-9. The first section makes an impelling argument for teaching substantial amounts of geometry in grades K-9: for practical life skills, for developing number and measurement concepts, for problem-solving experiences, for interest and fun, for developing intelligence, and for other reasons. These arguments can be used in curriculum committees examining the curriculum. The second section delineates the goals for geometry instruction. It includes content, awareness, and pedagogical goals. The last section, and by far the most extensive, delineates the scope, content, and sequence in geometry needed to implement the goals in the previous section. It is a blueprint, by grade levels, for the geometry curriculum, with specific illustrations of the content. (MNS)

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THE MICHIGAN COUNCIL OF TEACHERS OF MATHEMATICS

GUIDELINES FOR

QUALITY MATHEMATICS TEACHING

A CURRICULUM IN GEOMETRY FOR GRADES K-9

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A CURRICULUM in GEOMETRY for GRADES K-9

September, 1981

Report of the Geometry Curriculum Committee
Michigan Council of Teachers of Mathematics
May, 1980

Preface

The monograph presented here for ycur use is based upon the Report of the Geometry Curriculum Committee which was submitted to the Council Board in May, 1980. In October of the same year the MCTM Guidelines Committee decided to recommend the Report, with some modifications, to the Board as a monograph. In November, 1980 the Board approved the work as a monograph.

This monograph moderately breaks tradition with the typical MCTM monograph that presents classroom usable activities. This monograph rather, deals with the curriculum as it should be in an effective and complete mathematics curriculum in grades K-9. The Guidelines Committee, the Board and the authors considered this type material, and this topic, extremely important for classroom teachers, supervisors, and teacher education collegiate personnel to read, to discuss and to act upon.

The major part of the monograph is contained in three sections. In the first, an impelling argument is made for teaching substantial amounts of geometry in grades K-9. These arguments can be used in curriculum committees examining the K-9 curriculum or parts thereof. The second section delineates the goals for geometry instruction. It includes content, awareness and pedagogical goals. The last section, and by far the most extensive, delineates the scope, content and sequence in geometry needed to implement the goals of the previous section. It is a blueprint, by grade levels, for the geometry curriculum.

Even though the authors of this work consider it to be largely appropriate and correct, we realize that not every circumstance can be anticipated. Colleagues who find inconsistencies, errors, omissions, etc. are asked to bring them to our attention. We hope you find these pages useful to you as you consider your mathematics curriculum.

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A CURRICULUM IN GEOMETRY FOR GRADES K-9

GEOMETRY MUST BE TAUGHT IN GRADES K-9

Many teachers in grades K-9 (and in high school) have less than completely pleasant memories of their experiences with geometry. Mention the word "geometry", and "proof" comes to mind. It is understandable that K-9 teachers do not wish to teach proof oriented geometry to their students. However, the geometry this committee has in mind does not emphasize proof; rather it stresses intuition, informality, induction, discovery, observation and picture drawing. Student awareness of geometric ideas and relationships developed in an informal manner is exactly the type of experience the student needs to make high school geometry meaningful.

The first issue the geometry committee considered was:

WHY SHOULD WE TEACH GEOMETRY IN THE SCHOOLS?

This brief section is an attempt to share our arguments and illustrations.

For Practical Life Skills

The world abounds with examples of geometric ideas -- just look around! Forms are described geometrically as triangular, rectangular, perpendicular, parallel, congruent or similar. The learner needs to learn these ideas so that (s)he may communicate effectively. The argument can be made that all elementary mathematical ideas come from our interaction with the real world. Number is the mathematical abstraction developing out of numerousness in the world. Likewise, geometry is the abstraction developing out of size, shape and position in the physical world. It is just as important to learn about these ideas as it is to learn about numerousness.

Also in your daily life, you are often required to use geometric knowledge. For example, you may need:

a. to recognize a road sign by its shape.
b. to make a birdhouse or dress from a pattern.
c. to locate a lightswitch from a blueprint.
d. to order paint or grass seed to cover a specific region.
e. to determine the amount of concrete for a sidewalk.
f. to solve a geometric puzzle such as soma or a tangram.

The committee thus gives the following reasons for teaching geometry.

I. Geometric content is basic knowledge needed by most people to function effectively in everyday, home, work and play situations.

II. Geometry is the language used to describe spatial concepts and relations; its study promotes a deeper awareness and understanding of the real world.

For Development of Number and Measurement Concepts

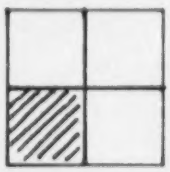
Many activities used to teach number and measurement ideas depend upon knowledge of geometric concepts for their understanding by the learner. Listed below are several typical examples of such activities. In the first illustration the learner should realize that the pencil is a physical representation of a segment and that the unit segment called a centimeter is being used to indicate how long it is. In the fourth, the learner is required to recognize that a unit region is divided into four equal parts and thus that one of them represents 1/4.

a. Measure your pencil in centimeters.

b. Find the area of the top of your desk in square decimeters.

c. Locate 2/3 on the number line.

d. Name the fraction represented by the shaded region of the diagram.



e. Illustrate that 2/3 is equivalent to 4/6 by shading a rectangular region.

f. Shade 45% of a square region.

g. Illustrate 0.57 on a hundred square.

h. Illustrate -3+7 = _____ on the number line.

Additional similar uses of geometry abound in the study of number and measurement. In each case the comprehension of the number or measurement situation depends upon geometric understandings. Thus, the committee argues that:

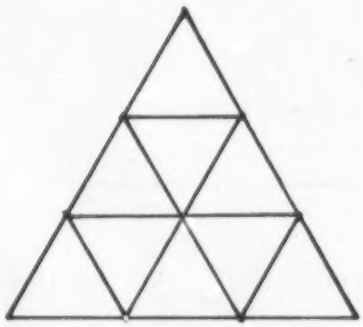
III. The knowledge of geometric forms and relationships is vital to the meaningful development of number concepts, algorithms and concepts of measurement.

For Problem Solving Experiences

A major objective of mathematics instruction is to help young people develop problem solving skills. Geometry plays two roles here: as a model for real problems like those in the previous paragraph and as a source of problems. It is the second role that is discussed here.

Geometry is a fertile source of simply stated and easily understood problems which can be investigated by students.

They range from simple instances such as "Count the triangles in Figure 1" to problems of finding properties of diagonals of quadrilaterals to sophisticated problems such as making a square with the same area as a circle. The variety and numerousness allows the teacher to provide problems at all ability levels. Thus, the committee argues:



IV. The study of geometry contributes to the attainment of important process goals such as exploring, conjecturing, testing and confirming or refuting hypotheses.

For Interest and Fun

Geometry, when approached informally and intuitively, can be fun and interesting for students. At the elementary and middle school levels, especially, most geometric work should be based on physical objects and good models or drawings. Students can then seek relationships and properties by using materials which they can cut, draw, paste, etc. These activities are appealing to children. Thus, the committee argues:

V. The study of geometry is interesting and fun and provides a good change of pace.

For Development of Intelligence

Psychological and medical researchers have recently found that the halves of the brain have different major functions. One half is spatially-figurally oriented, while the other half is more symbolically oriented. Reading and arithmetic are mainly processed by the symbolic half; geometry by the other. To maintain a balanced intellectual development, both halves need to be regularly employed. Moreover, since geometry is visual, tactile and concrete, it is an appropriate area for study by young children. Thus, the committee argues:

VI. The study of geometry is psychologically and developmentally sound.

For Other Reasons

The committee suggested many additional arguments for including geometry in the K-9 curriculum. Some of them were:

VII. Geometry in K-9 provides a necessary basis for the study of formal geometry in the high school.

VIII. Geometry is an important part of our cultural heritage.

IX. Geometry provides the basis for appreciating many art forms.

GOALS FOR GEOMETRY STUDY IN GRADES K-9

The following goal statements were prepared and are presented under the premise that

Geometry is a basic skill equal in importance to computation.

Content Goals for Students*

1. Distinguish among 1-, 2- and 3- dimensional shapes.
2. Identify by name, common 1-, 2- and 3- dimensional shapes in the physical world and in diagrams.
3. Know and apply the basic properties of common 3-dimensional shapes.
4. Construct models of common 2- and 3-dimensional shapes.
5. Draw common 1-, 2- and 3-dimensional shapes in two-space.
6. Know and apply the basic properties of quadrilaterals, circles, and triangles.
7. Know and apply the concepts of parallelism, perpendicularity, congruence, similarity and symmetry.
8. Perform basic constructions such as reproducing an angle.
9. Interpret and make scale drawings.
10. Illustrate and use the topological concepts of inside and outside.
11. Identify and illustrate the properties of figures under a translation, rotation, reflection, half turn, and size transformation.
12. Find the measures of 1-, 2- and 3-dimensional shapes and of angles to a specified degree of accuracy.
13. Choose an appropriate unit of measure in a given measurement situation.
14. Estimate the measures of 1-, 2- and 3-dimensional objects within appropriate limits.
15. Specify position using coordinates, e.g. on a map.

*The order of these goals does not indicate order of importance, --all are important.

16. Construct and interpret graphs.

17. Use informal reasoning in geometric situations.

18. Select and use appropriate geometric models in problem solving situations.

19. Select and use appropriate geometric models of number concepts.

Awareness Goals for Students

1. Become aware of the role of geometry in daily life, e.g., in career, consumer and recreational situations.

2. Become aware of the role of geometry in nature and in the development of world culture.

3. Become aware of the usefulness of geometric ideas in illustrating other mathematical ideas.

Pedagogical Goals for Teachers

1. Help youngsters develop skill in spatial perception (visualization).

2. Help youngsters develop rational thought processes.

3. Help youngsters recognize geometry as a mathematical model of the world.

4. Help youngsters develop problem solving skills.

5. Recognize that teaching geometry in grades K-9 should emphasize intuition, informality, induction, discovery, observation and picture drawing.

6. Provide sufficient time and materials for students to work informally with geometric concepts--DELAY ABSTRACTION!

7. Provide sufficient opportunities for the study of geometry so that students may successfully study additional mathematics and other subjects with geometric prerequisites.

8. Help youngsters recognize that geometric knowledge contributes to the understanding of other mathematical ideas.

GEOMETRY IN GRADES K-9

Basic Principles

These suggestions for the sequencing of and the nature of the content in geometry for the K-9 mathematics curriculum are based on the following principles:

1. A person's intellectual development with respect to geometry progresses from the gross recognition of real world models of geometric objects to an organized body of geometric knowledge about which the person can reason.

2. The development of geometric knowledge in a person is related to the amount of and to the nature of the instruction in geometry provided. That is, fully developed understanding of geometry does not occur as a result of maturation alone.

3. Instruction should use everyday three-dimensional objects and their characteristics to introduce one and two dimensional concepts whenever possible.

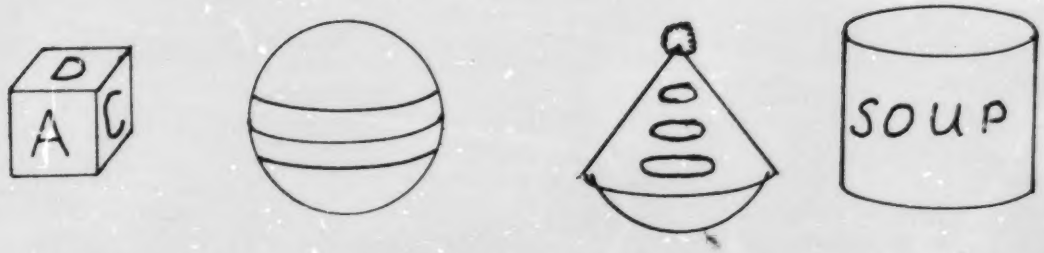
4. Geometric learnings should include concepts (e.g. triangle, parallel, similar), skills (e.g. calculating areas and perimeters, constructing figures, sketching solids), and applications (e.g. reading maps, using scale drawings, buying paint for a room).

5. Geometry should be integrated into and used in the learning of other mathematical material (e.g. picturing fractions, graphing equations, using objects in number work) in the elementary and middle school curriculums.

Content and Sequence by Grade Level

KINDERGARTEN - GRADE 1

SPACE SHAPES (SOLIDS) Three-dimensional (or space) shapes should be introduced through examination of everyday objects. The following shapes should be included: cube (block), box (rectangular parallelepiped--cardboard box), sphere (ball), cone (party hats, sugar cone), and cylinder (tin can).



The first concern should be recognition of different space shapes (solids). They should be sorted by shape and, within a shape category, by size. Practice recognizing these shapes in different orientations in space and from different perspectives (side, top, bottom, etc.). These shapes can also be examined for properties such as "stackability", "rollability", and "space filling ability" -- Can you pile one on another? Will the shape roll across the floor? Which shapes can you use to build a wall?

INSIDE, OUTSIDE

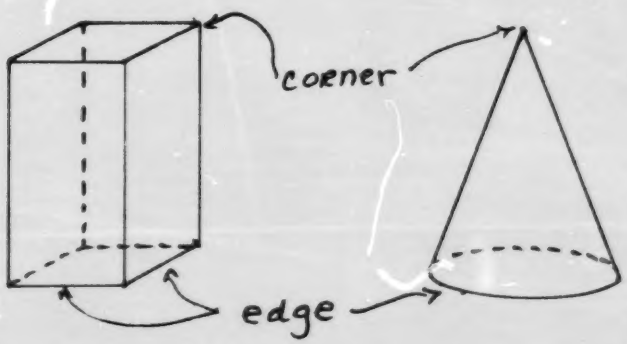
Introduce INSIDE and OUTSIDE -- A coin inside a space shape rattles when the shape is shaken; one outside does not.

CUBE, BOX,
CONE, CYLINDER

Names should be learned: CUBE, BOX, CONE, CYLINDER.

After familiarity with the space shapes examination of their characteristics should begin. Emphasize similarities and differences. Identify FACE, EDGE (where faces meet), and CORNER (where edges meet). Which shapes have corners? Are they sharp or blunt? Which shapes have flat, curved or rounded faces? Which have edges? Are the edges straight or do they turn? Sort shapes on basis of attributes such as flat face, curved edge, rounded face, number of corners, etc.

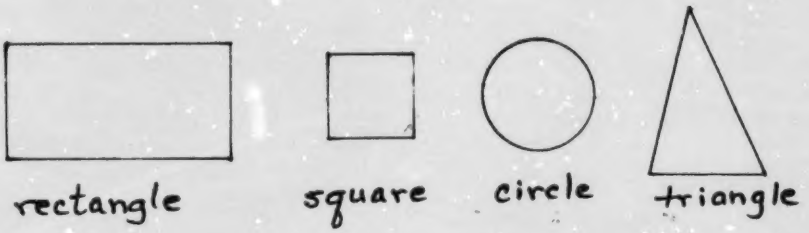
FACE
EDGE, CORNER



PLANE SHAPES

Introduce plane (flat) shapes by tracing faces of space shapes. An alternative procedure is to outline shapes with masking tape (or draw a brick wall). Get rectangle, square and circle. Get triangle by cutting a rectangular face in two parts. Emphasize identifying different shapes in the world and in pictorial material. ASK, "What shape is _____ like?", where "window pane", "clock face", "sail", etc. can go in the blank. Identify shapes in different positions in the plane, i.e., tilted or turned. Learn names: RECTANGLE, SQUARE, CIRCLE, TRIANGLE.

RECTANGLE, SQUARE, CIRCLE, TRIANGLE

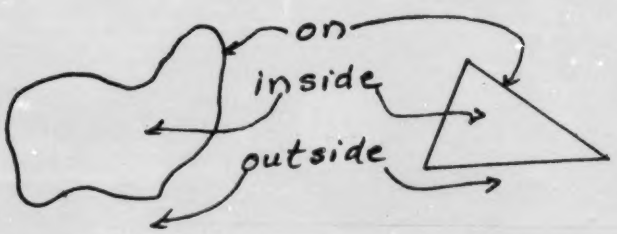


PATH

Introduce the idea of PATH between two locations on the plane. What is the path from your seat to the door? the toy box? the coat hanger? Introduce the special path called the SEGMENT or LINE SEGMENT (sometimes LINE[1]). Notice that plane figures are made up of paths, (sometimes these paths are segments) that start and end at the same spot. You use a path to draw a pond and you are wet if you are inside the pond, but dry if outside. Find spots (points) inside, outside and on plane shapes.

LINE SEGMENT

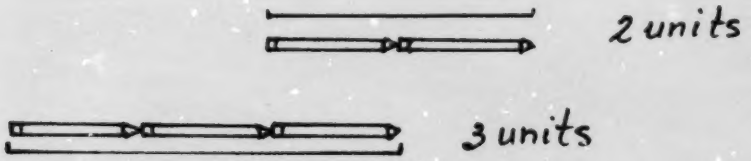
INSIDE
OUTSIDE
ON



[1]The word "line" has a variety of meanings in English. It may mean the "row of words in a reading book or the queue you stand in to check out at the grocery store. It is preferable, but not absolutely necessary, to use "line segment" for straight paths with beginning and ending points rather than "line".

MEASUREMENT

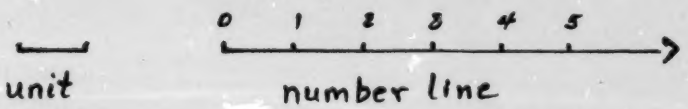
Introduce linear measurement by placing coffee stirrers (chalk, pencils, paper clips, feet) or other "segment-like" objects end to end along a longer segment. Describe the longer segment in terms of the number of coffee stirrers (or other units).



UNIT

NUMBER LINE

The basic idea is to make a replica of a given segment by using "unit" segments. Use "unit" segments which are the same length. Begin the idea of the number line by numbering the right hand end points of segments measuring another segment.



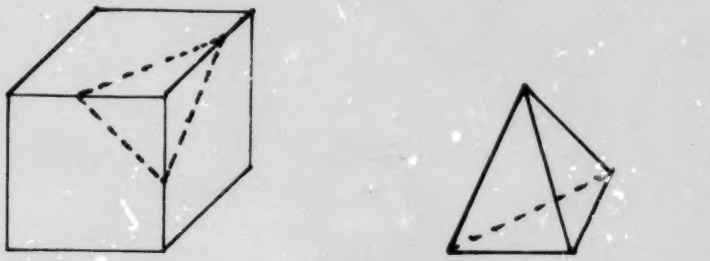
COORDINATES

Informally introduce coordinates to students. Locate chairs in the room by row and seat -- row 3, seat 5 etc. Extend to seats in auditorium or in movie house.

GRADES 2 and 3

PYRAMID

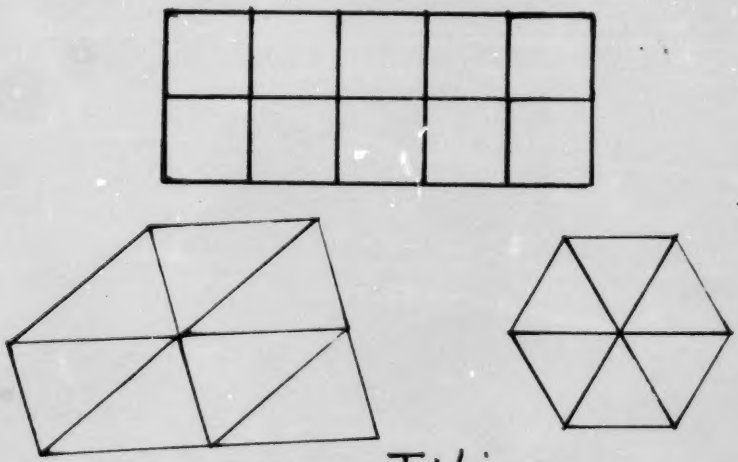
Review space figures and add the pyramid. The pyramids of Egypt are good models from social studies. Pyramids with square (or rectangular) bases and triangular bases are sufficient. Cutting the corner off a box gives a pyramid. Learn the name, PYRAMID.



Pyramid with triangular base

TILE
CORNERS: SQUARE, SHARP, BLUNT

Continue familiarity work with plane shapes. For example, determine which of the plane shapes can be used to cover (tile) a plane surface such as a floor or table. Introduce the square corner, sharp corner and blunt corner for plane figures. Begin to recognize that 4 squares or 4 rectangles, because of their square corners, fit nicely at a point, but more triangles need to be used because each angle (corner) is needed twice.



Tilings

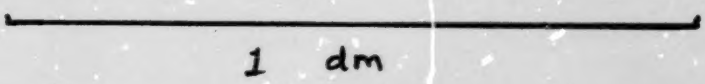
Identify the corners and sides of plane shapes. Classify plane shapes by the number of sides. Sort triangles on the basis of some attribute such as the nature of the corners (blunt, sharp, square) or the length of two sides. Compare sides of figures by length. Order squares and circles by size. Find examples of objects in the world that look like segments, triangles, rectangles, etc.

RULER

Fasten models of segments (such as paper clips) together to form a ruler. Introduce rulers in whole unit subdivisions only. The ruler is an instrument representing several unit segments in a chain.

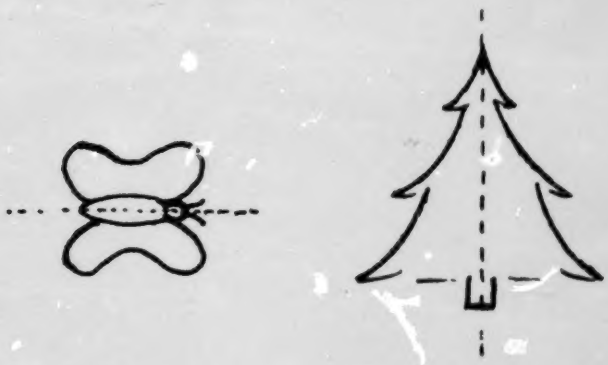
DECIMETER
METER

Introduce standard units of DECIMETER and METER (in that order). Practice measuring in dm and m using dm and m rulers with no other subdivisions. (A piece of string 1 dm long or 1 m long is often quite useful to use as a ruler.)



FOLDING SYMMETRY

Introduce folding symmetry for geometric figures. The two parts of the figure are exactly alike--they exactly fit one over the other when the shape is folded into two pieces.



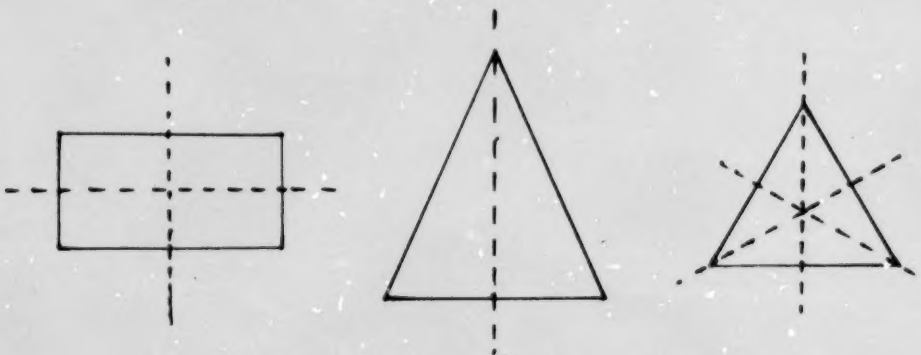
Relate to science by noticing hearts have folding symmetry as do butterflies,

PROPERTIES OF PLANE SHAPES

beetles, spiders, leaves, etc. Examine rectangles, squares, circles, and triangles for folding symmetry. Seek recognition or confirmation of characteristics of plane shapes based upon folding symmetry. For example, since a pair of opposite sides of a square fold on top of each other, they are equal in length; the point on all symmetry folds for a circle (disc) is the center of the circle; two square corners on the same side of a rectangle are equal, etc. In late third grade introduce the ISOSCELES TRIANGLE as one with a single symmetry fold, and the EQUILATERAL TRIANGLE as one with three symmetry folds.

ISOSCELES TRIANGLE

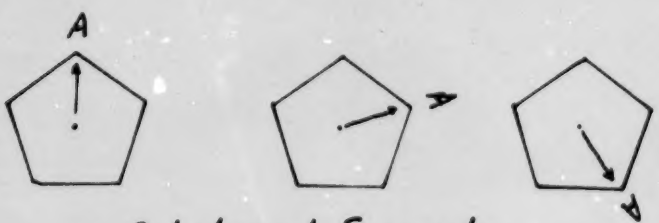
EQUILATERAL TRIANGLE



Symmetries of plane shapes

Characteristics of plane shapes can be investigated by trying to fit them into holes of the same shape. Try to fit a rectangle, square, circle and different triangles into their outlines without turning the shape over. Determine how many different ways a shape will fit its hole (without turning it over). Show how some shapes will fit more than once by "turning the shape about a point" --this is "turning symmetry (or rotational symmetry)". Characteristics of the shapes can again be highlighted. The above activities could be nicely summarized as a bulletin board display.

TURNING SYMMETRY

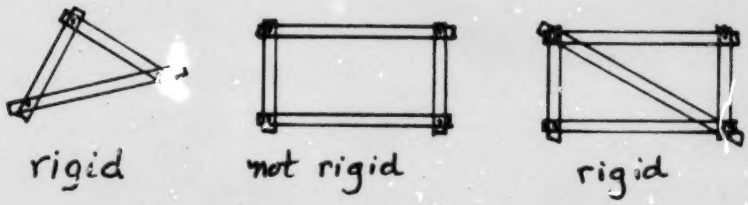


Rotational Symmetry

RIGID FIGURE

DIAGONAL

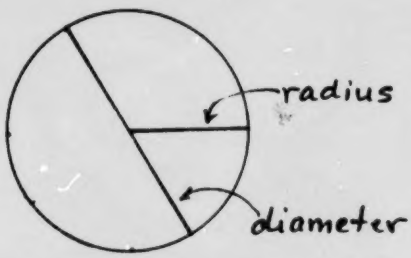
Explore the rigidity of plane shapes. Make shapes of cardboard strips fastened at ends by paper fasteners. Notice that some shapes change when picked up--they are not rigid. Notice that a triangle does not change shape--it is rigid. Explore "adding diagonals" to non-rigid shapes. When shape is dissected into triangles by diagonals it becomes rigid. The use of triangles in architecture would make a nice bulletin board display.



CENTIMETER

DIAMETER
RADIUS

Introduce the CENTIMETER (cm) as a unit of measure. Find a body part such as thumb or finger with width about one centimeter. Use the centimeter to measure paths (to the nearest cm with cm rulers). Measure the sides of plane shapes (square, rectangle, triangle)--again confirming characteristics of sides. In addition, measure diagonals of polygons and diameters and radii of circles. Also measure edges of three dimensional objects -- describe in terms of the lengths of the edges, etc.

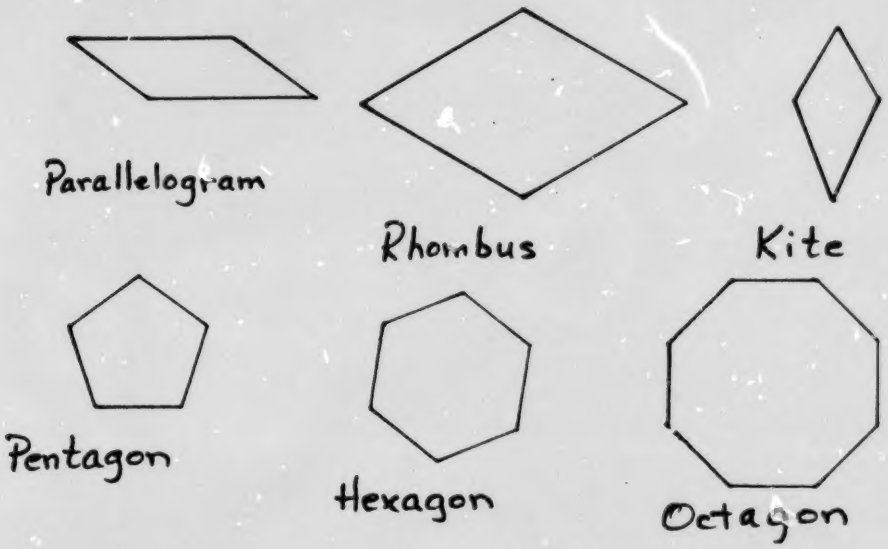


PARALLELOGRAM
RHOMBUS, KITE,
PENTAGON

Introduce additional plane figures found in the environment. Find examples of parallelogram (squished rectangle) rhombus (squished square), kite (like the flyer) and regular pentagon (5) (the building in Washington D.C. or the darkened shapes

HEXAGON
OCTAGON

on a soccer ball), hexagon (honeycomb) and octagon (stop sign). Emphasize recognition of shape and giving that shape its name: PARALLELOGRAM, RHOMBUS, KITE, PENTAGON, HEXAGON, OCTAGON. Show shapes (not pentagon or octagon) in tiling patterns, in art and in nature. Do some informal folding and turning to see which have symmetry and which do not.



CUTTING SYMMETRY

Introduce symmetry of space figures with respect to a plane (cutting symmetry). Cut solids such as cubes, boxes, balls, cones, cylinders and pyramids to see if they can be cut into two identical parts. Build on the common experience of cutting an orange or an apple into two pieces for sharing.

NETWORKS

Introduce NETWORKS in the plane as part of Map Study. Choose three or four cities on a map and highlight the roads connecting the cities (this is a network). Kids can determine the number of paths (roads) from one city to another and their distance. Can a traveling salesperson visit each city without taking the same road twice? Can the road inspector travel each road without wasting fuel by going over one more than once? Make straw models of solids. These are

SKELETONS

SKELETONS. Ask the same kind of questions as with networks.

Network of Roads

Skeleton of Pyramid

GRADES 4 and 5

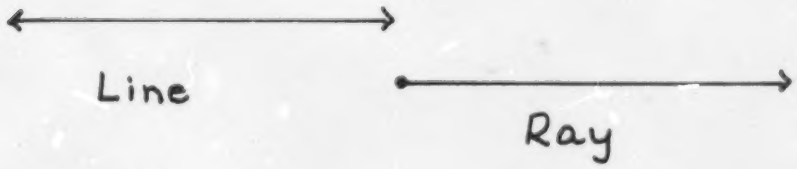
Review plane and space shapes introduced in previous grades to insure student readiness to use those shapes in other activities.

LINE

Introduce LINE as a "stretched line segment"--one that goes on and on in two directions. Telephone lines, fishing lines and kite strings are imperfect models of lines (no perfect models can be found because lines are infinite). Help kids use their imaginations to imagine lines in their minds.

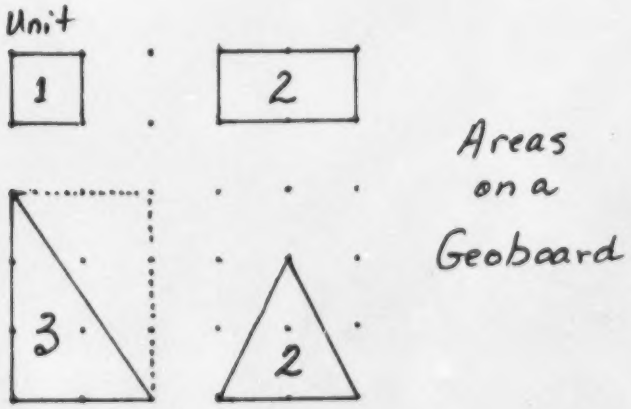
RAY

Introduce RAY as part of a line. It begins at a point and goes on forever in only one direction. Moonbeams, spot lights, flashlights, sun rays are good models. Again get kids to use their imaginations.



AREA

Introduce AREA of common figures such as square, rectangle and triangle. Have students use bits of paper, tiles, etc. to cover common figures. When the bits of paper or tiles are all alike and none overlap, the number of them used is the area. Use GRIDS to simplify this process later. Use geoboards (or dot paper) to help kids see how to "count" area by counting the squares enclosed by a shape.



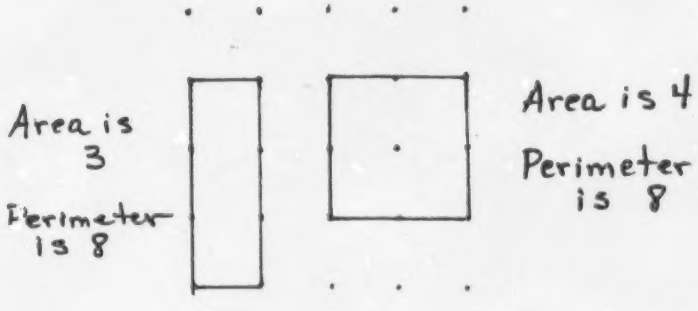
Discuss the "square" as the easiest unit to use. It is easy because the number of squares that fit along a side of a shape equals the length of the side. Do not teach formulas here! Wait until kids have firm understanding that the area is the number of units needed to cover a shape, then introduce formulas.

PERIMETER

CIRCUMFERENCE

NEAREST UNIT

Introduce PERIMETER as the length of the path making up the plane shapes of square, rectangle, parallelogram, triangle, and circle (for circle the perimeter is called circumference). Measure with whole unit rulers. Use the unit of cm, dm and m. Measure many common objects in environment. Help students choose most appropriate unit of measure for a given object. Practice measuring to the "nearest" unit. For the rectangle and square show that figures with a given perimeter may have many different areas. Use string for this type activity. Emphasize that a shape of given perimeter can have many different areas and vice versa.



Same Perimeters -- Unequal Areas

NONAGON, DECAGON, POLYGON

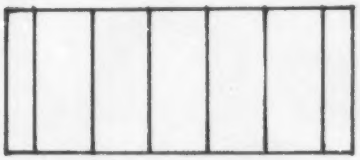
Introduce additional regular figures such as NONAGON (9 sides), DECAGON (10 sides), POLYGON (many sides). Develop schemes for naming regular polygons by prefixes e.g. octa, nona, deca, poly, etc.

PARALLEL

PERPENDICULAR

Introduce two new relations between lines: PARALLEL and PERPENDICULAR. Parallel lines are two lines in a plane that do not cross; they go in the same direction like the sides of a straight ladder or the yard lines on a football field, or the mortar joints in a brick wall. Perpendicular can be thought of as making

a "square corner" (in the plane). Relate to carpenter square or draftsmen's square. Find examples of perpendicular lines.



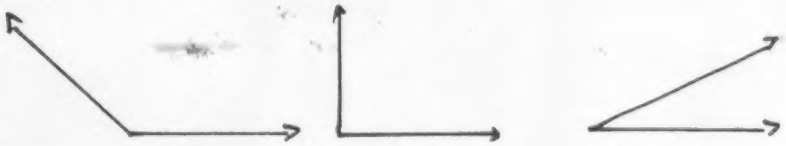
Parallel Yard Lines
Yard Lines Perpendicular to Side Lines

ANGLE
OBTUSE, RIGHT, ACUTE

Name corners of plane figures as ANGLES. Compare blunt, square and sharp corners. Use a dictionary to find synonyms: OBTUSE, RIGHT, ACUTE. Use these names for angles. Identify, as obtuse, right and acute, the angles of the plane shapes known.

RIGHT TRIANGLE

Name the triangle with a right angle as the RIGHT TRIANGLE.



Obtuse Right Acute

CONGRUENT FIGURES

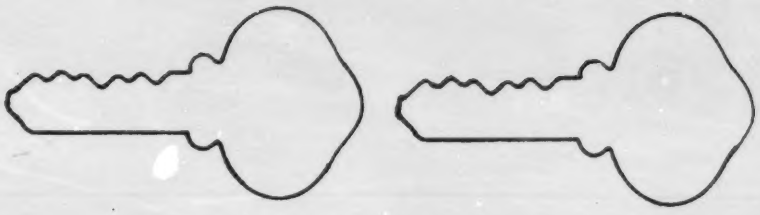
Develop concept of CONGRUENT FIGURES as those that are exactly alike--in the plane the figures could be made to fit one on the other. Practice identifying congruent and noncongruent figures. Use models and faces of models as well as real objects such as student desks,

CORRESPONDING PARTS

paper clips, etc. Introduce CORRESPONDING PARTS of congruent figures by fitting figures on each other. The corresponding sides and angles fit over each other and are equal. Examine congruent figures that are not common figures, i.e. squiggles, curves, broken lines, photo copies, student desks, auditorium seats, new crayons, etc. Relate congruence to

EXACT COPIES

ASSEMBLY LINE, EXACT COPIES OF KEYS, DUPLICATE PARTS, etc.



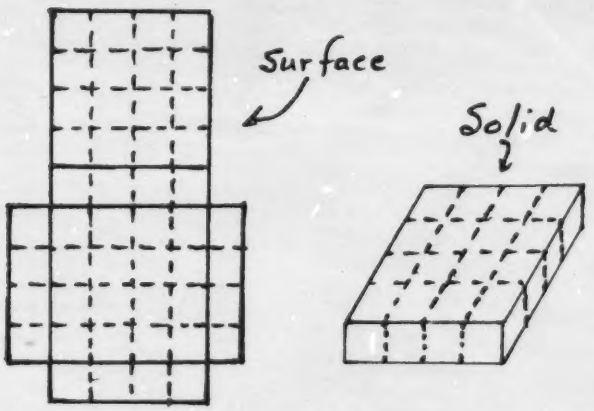
Exact Copies

REGULAR POLYGONS

For REGULAR POLYGONS, show that corners all are congruent as well as sides. Investigate relation between congruence and area and perimeter. Near the end of the work on congruent shapes introduce some common calculating formulas for finding areas: Area of square = side x side = s x s; Area of rectangle = base x height = b x h; Area of right triangle = half the base times the height = ½xbxh.

SURFACE AREA

Apply the calculation of area to SURFACE AREA of cubes, boxes, and pyramids. (Congruent shapes are used extensively in fraction work, and area of a rectangular region forms the basis for teaching multiplication of fractions.)



Area of the Surface of Solid

PROPERTIES OF PLANE SHAPES

Examine properties of the plane figures: parallelogram, rhombus, kite and regular polygons. Use folding symmetry and rotational symmetry to find congruent parts of plane shapes, congruent diagonals,

perpendicular diagonals, parallel sides. Determine the number of diagonals that can be drawn from any one vertex in a polygon. If all the diagonals from one vertex are drawn, how many triangles are formed? Students should use models to investigate these figures--properties or characteristics should be summarized.

RELATIONS AMONG SHAPES

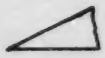
When plane shapes and their properties are recognized by students, it is time to extend their understanding to relations among shapes. For example, since squares and rectangles have four right angles, they are related (if you drew several four sided polygons with all right angles you would probably get both squares and rectangles). Are squares also rectangles? Are rectangles also squares? Common properties of different shapes can be found in various ways--using symmetry, measuring, observing, or by giving characteristics of a quadrilateral (four sided shape) and drawing shapes meeting these characteristics. For example, ask student to draw a four sided figure with the following characteristics: It has four right angles. The results will include squares and rectangles. As another example ask: Draw a quadrilateral with two pairs of opposite sides parallel? The results will include parallelograms, rectangles, squares and rhombi. These activities illustrate that certain characteristics determine classes of quadrilaterals--thus the quadrilaterals are related. Activities such as those suggested above illustrate the way DEFINITIONS are used in mathematics (certain conditions always produce the same results). The activities also provide chances for informal deductive reasoning.

QUADRILATERAL

DEFINITIONS

ANGLE MEASURE

Introduce students to measuring angles. Begin with a set of congruent angles (cardboard wedges) to be used as a "unit" angle and a set of other angles to be measured. Students use unit angles to "fill" the other angles with no overlapping and all vertices at the same point. The measure is the number of unit angles used.



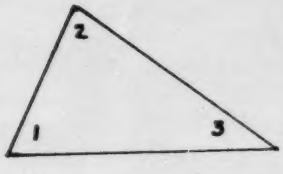
Unit Angle

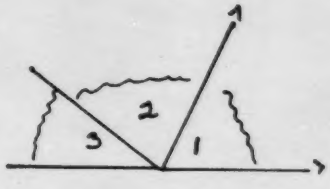
Angle of Measure 3

PROTRACTOR

DEGREE

Repeat with different, larger or smaller, unit angles. Use the unit angles to make a protractor--place several wedges around a point and mark the edges and the point. Use this instrument to measure angles. Introduce the standard protractor in which the unit angle is called a DEGREE. Measure acute, right and obtuse angles. Redefine or describe these angles in terms of degree measures: less than 90°, equal to 90°, greater than 90°. Investigate angle sum of angles of a triangle by tearing corners off paper models and fitting them around a point.





Confirm observations by measuring angles of a triangle with protractor and adding results. Note that often the sum is not 180° exactly. Discuss the approximate nature of measurement. (Computing the mean of the sum of the angles of several triangles usually gives a value near 180°.)

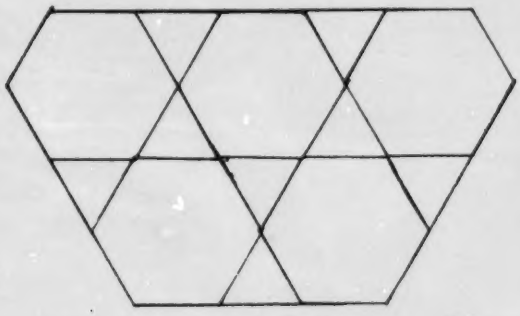
MEASURING TURNS

Describe turns (rotations) in terms of the degree measure of the angle of the turn. A doorknob is a nice model for

turns. So are the hands of a clock. A quarter turn is a rotation through 90°; a half turn is a rotation through 180°. How many degrees in an eighth turn? (These activities provide practice in using fractions.)

TESSELLATIONS

Tiling patterns (TESSELLATIONS) are a valuable resource to help students observe or discover properties of figures. Have students construct "floor tilings" using 1) triangles, 2) quadrilaterals (all will work), 3) other selected polygons. Notice how the polygons must fit at a point to completely fill the region around the point with no overlapping. Look for angle sum relations. Also look for parallel lines relations.



Triangle-Hexagon Tiling

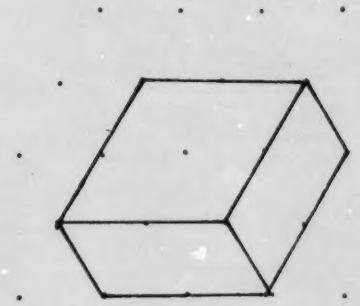
Relate to art by coloring patterns, by observing Escher patterns, by looking at wall paper designs and by looking at Moorish ceramic work. Many patterns can be formed by sliding a shape repeatedly. Such patterns have SLIDE SYMMETRY. Investigate various ways a pattern can be repeated on a strip (frieze patterns). See if you can find some frieze patterns involving half turns and slides.

SLIDE SYMMETRY

Frieze Pattern

SKETCHING SHAPES

Begin drawing sketches of three dimensional objects in the two dimensional plane. Begin with simple figures like the cube and box. Help students visualize the square or rectangular tops as parallelograms. Help them put in the "hidden" edges as dotted lines. Practice "drawing what you actually see". A very useful tool in drawing is "isometric dot paper" (dots at vertices of equilateral triangles).



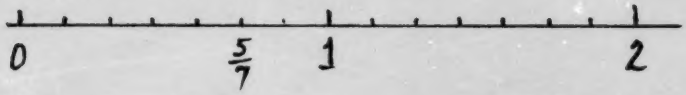
Box Sketched on Isometric Dot Paper

Sketching may be helped substantially by examining shadows of squares and rectangles when the light source is thought of as located at the position of the eye and the light shining perpendicularly to the surface on which the shadow falls. Cylinders with circular tops can also be sketched.

NUMBER LINE AND FRACTIONS

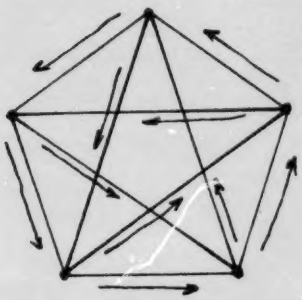
Earlier the number line with whole number points was introduced. Here partitioning with fractions is appropriate. The key to understanding the "rational" number line is in the understanding of the fraction concept as a measure of a part of a whole. On the number line the fraction 3/4 is placed at the end of 3 of four congruent subdivisions of a unit segment. Thus it represents a point 3/4 of the distance from 0 to 1. Similarly, x/y represents a point $\underline{x}$ units, of length $\underline{1/y}$ each, from the initial zero point. For example, 5/7 is placed at the end of the fifth of

seven equal subdivisions of the unit segment. The number line can also be related to scales on rulers--both metric and conventional.



NETWORKS

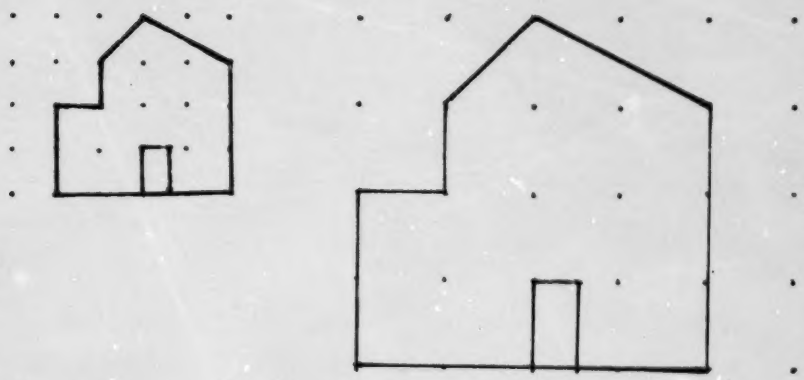
Draw networks related to transportation and geography. Determine whether networks are traversable without retracing any paths. Look for generalizations about vertices with an even or an odd number of paths at them. Look also for relations between the number of vertices, paths and regions for a given network. (Regions + Vertices - 2 equals Paths)



Retracing a Path is not Allowed! Can You Find Another Way?

SIMILAR

Introduce SIMILAR figures through scale drawings and models. Many model airplanes and cars are exact replicas of the real things, but smaller. Illustrate scaled figures that are larger and smaller than the original. Biology is a good source of scales that result in larger figures. Use squared paper (or dot paper) to help youngsters draw figures similar to ones given to them on squared paper (or dot paper) with a different size unit square. Draw corresponding parts by connecting easily identifiable corresponding points.



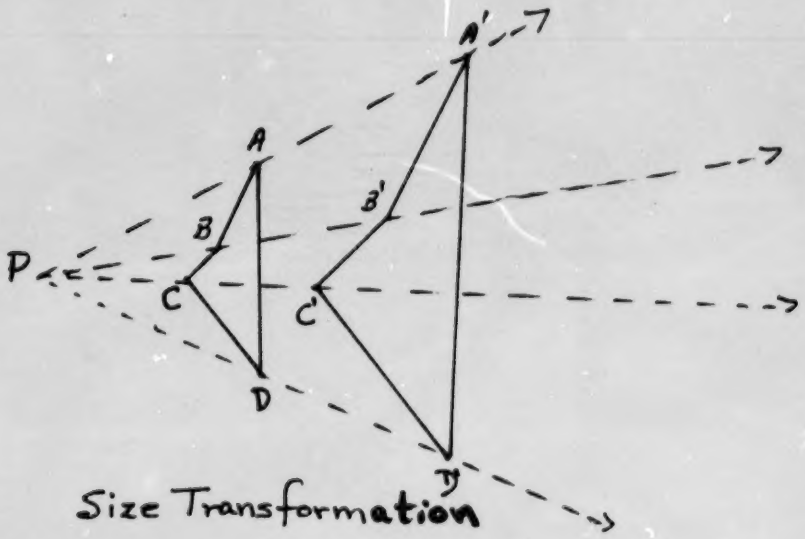
Similar Figures using Dot Paper

SIZE TRANSFORMATION

Identify examples of similar shapes--picture vs real object, real object vs model, etc. Introduce drawing similar figures by a SIZE TRANSFORMATION: 1) Choose a point P in the plane. 2) Choose a multiplier M. 3) Choose several points on the figure given. 4) Draw rays from P through the points on the figure. 5) Measure the distance along a ray from P to a point on the figure. Call this distance r. 6) On that ray find the point that is M x r units from P. 7) Repeat for all points and connect with paths. The result is a figure, A'B'C'D', similar to the original, ABCD. Point out that corresponding distances are related

MULTIPLY RELATION

by the multiplier M. This multiplicative relation is an essential idea for similar figures.



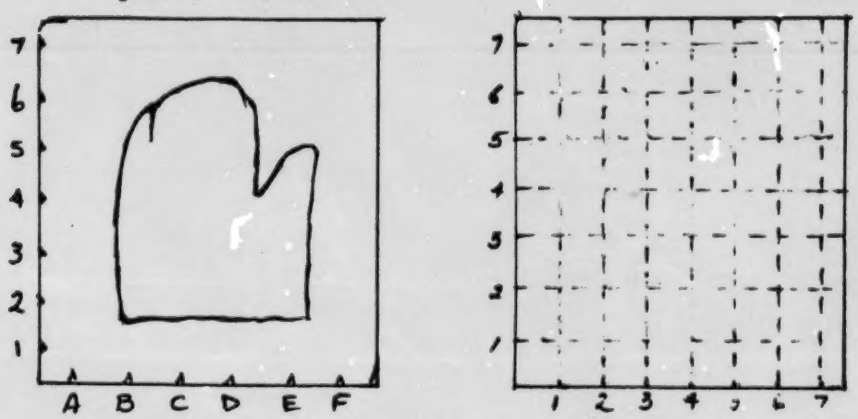
RELATING SHAPES

Extend the ideas of relating plane figures and the role of definition in geometry. Notice, for example, that a quadrilateral with two pairs of parallel sides is always a parallelogram and sometimes it is also a rhombus, rectangle or square. Thus rectangles, squares and rhombi are also parallelograms. More conditions are needed on the parallelogram to ensure getting a rectangle, for instance. This is informal deduction! Classify plane shapes by their symmetries--turning and folding. Note that certain symmetries lead to properties of the shape. For example, a quadrilateral with half-turn symmetry has opposite sides parallel and congruent; one with quarter-turn symmetry has adjacent sides congruent, etc. Point out differences as well as likenesses among shapes.

RECTANGULAR COORDINATE SYSTEM

Begin the development of the notion of a RECTANGULAR COORDINATE PLANE. One nice tool that could be used is the map of your state. There the coordinates are given by a number and letter pair.

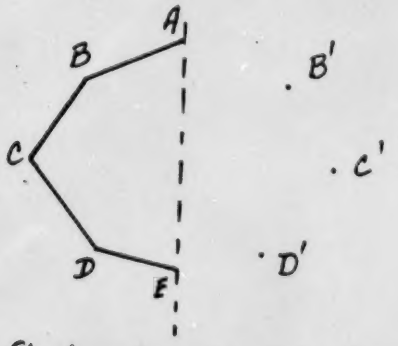
Find the cities, historical sites, airports, etc. using these coordinates. Make up coordinate systems using East and North directions, i.e., find the point 4E, 5N.



Map and Coordinate Systems

Finally, move to first quadrant ordered pairs. Good practice here is drawing pictures given as coordinate sets or playing battleship! Use rulers to connect points with segments to help students see that two points determine a segment and a line. Additional practice with the coordinatized plane can be achieved by having students find the coordinates of the images of simple figures under quarter and half turns and under translations. Introduce REFLECTION WITH RESPECT TO A LINE in terms of folding symmetry.

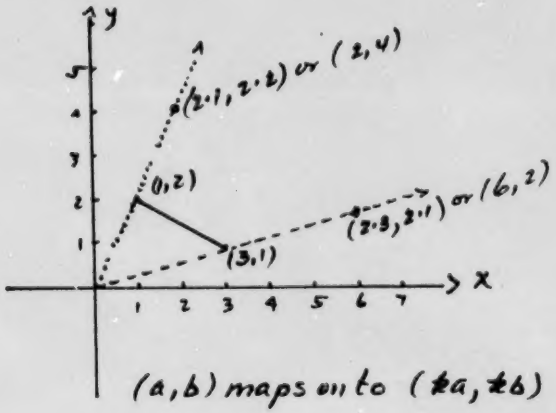
REFLECTION



Reflecting Line is Fold Line

SIZE TRANSFORMATION

Give coordinates of points reflected over horizontal and vertical lines. Similarly find the coordinates of image points under an enlargement or reduction (size transformations) when the origin is the inicial point. Help children discover the multiply relation: (a,b) maps onto (ka, kb) when k is the multiplier for the size transformation. Draw similar figures in the coordinate plane using the relationship above.



POLAR COORDINATES

BEARINGS

Locate points in the plane by giving the distance from the origin and measure of the angle formed by the point, the origin and the positive x-axis. These are POLAR COORDINATES. Use polar coordinates to locate points outside the first quadrant. Introduce BEARINGS as the measure of the angle clockwise from due North. Relate to navigation.

PROPERTIES OF PLANE SHAPES

PARALLELISM

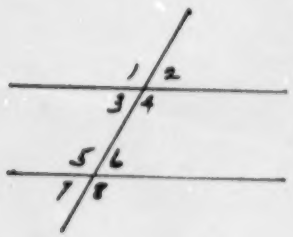
Introduce students to the properties of CIRCLES, ISOSCELES TRIANGLES, KITES and RHOMBI. Much of this can be done by folding paper or light cardboard models of the figures. For a circle all symmetry folds contain the center, a non-symmetry fold (chord) is bisected perpendicularly by a symmetry fold, diameter equals two radii and center is equidistant from all points on the circle. Similarly, folding can be used to illustrate properties of isosceles triangles, kites and rhombi. As properties are found and agreed to, list them for all to see and use. PARALLELISM was introduced earlier, but the results

of parallelism were not developed except as they related to parallelograms. Some formalization of the relations between angles formed by a line intersecting two parallel lines is appropriate.

ANGLES AND PARALLELS
CORRESPONDING
INTERIOR
EXTERIOR
ALTERNATE

CORRESPONDING, INTERIOR, EXTERIOR, and ALTERNATE angles should be introduced and their relations noted. A model consisting of 3 wooden strips and two circular protractors provides an effective way for noting special angle relationships formed by parallel lines. This material is also especially appropriate for informal deduction. As properties are observed, students should develop arguments to support their observations. Congruent angles have the same measure. Parallel lines should be sketched by drawing appropriate congruent angles using the protractor.

SKETCHING PARALLELS

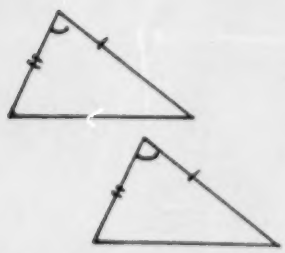


Parallelism Guaranteed by
$\angle 1 \cong \angle 5$, or
$\angle 4 \cong \angle 5$, or
$m\angle 4 + m\angle 6 = 180°$

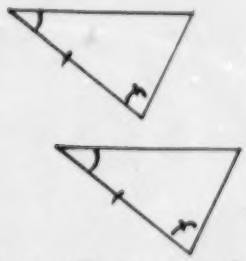
CONGRUENT TRIANGLES

Some conditions which produce CONGRUENT TRIANGLES can be introduced. In particular, SAS, ASA and SSS. Students should be shown how to replicate a segment using a compass and how to draw congruent angles using a protractor (or compass, if you wish). The results of drawing two triangles with SAS equal should be shown to be congruent by superposition. Similarly for SSS and ASA.

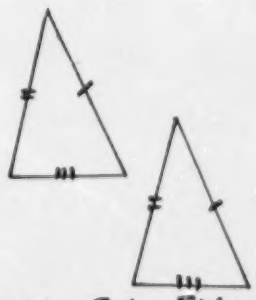
SAS
ASA
SSS



Side-Angle-Side (SAS)



Angle-Side-Angle (ASA)

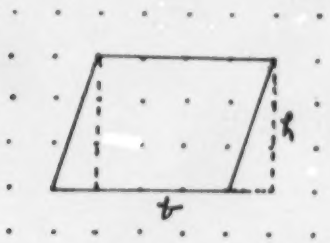


Side-Side-Side (SSS)

Conditions which guarantee Congruent Triangles

AREA FORMULAS

The area formula for the PARALLELOGRAM should precede those for the CIRCLE and the TRIANGLE. The parallelogram formula can be developed out of that for the rectangle. The geoboard nicely illustrates this.



Area = b·h

The circle formula can be developed by cutting two semi-circles into an equal number of pie shaped wedges and fitting them together to approximate a parallelogram. The triangle is half a parallelogram and thus its area is half the base times the height of the parallelogram. This suggests that the formula for the parallelogram and rectangle ought both to be "base x height" (not length x width for rectangle). As an application of area, construct squares on the three sides of a right triangle. Look for the relationships between the sum of the areas of the two smaller squares and the area of the largest square.

PYTHAGOREAN RELATION

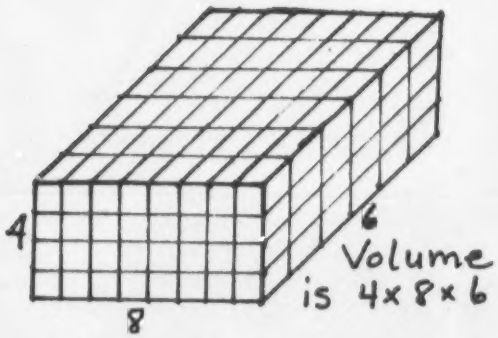
This is the PYTHAGOREAN relation: $a^2 + b^2 = c^2$. Further apply area formulas to find the SURFACE AREAS of solids.

SURFACE AREA

Surface area can be introduced by unfolding paper models of solids to see what plane figures make up the surface.

VOLUME

VOLUME can be viewed as the measure of capacity--how many cubes are needed to fill a space shape. The formulas for volume can be viewed as short ways of counting these cubes. The simple formulas for a CUBE and a BOX should be developed.



INVARIANTS

Investigate the properties of plane figures that change or do not change when transformed by 1) a slide, 2) a rotation, 3) a reflection, 4) a size transformation. The unchanging properties are called INVARIANTS. Investigate further the invariants when you compare a shape to its shadow or a figure to one on a rubber sheet that is stretched.

COORDINATE PLANE

Extend the rectangular coordinate plane to the full four quadrants. Plot points with negative x and/or y coordinates.

PLANE SHAPES

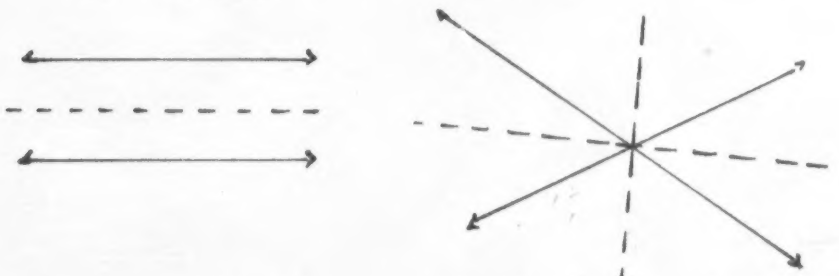
TRANSFORMATIONS

Review plane shapes by drawing shapes given the coordinates of the vertices and vice-versa. Review transformations--reflection, rotation, slide, size--by transforming shapes in the coordinate plane. For reflection use horizontal and vertical lines and the lines $y = x$ and $y = -x$; for rotations and size transformations use the origin as center. Look for patterns relating preimage coordinates to the corresponding coordinates of the image.

CONSTRUCTION

Introduce basic constructions of geometric figures--the angle, the segment, the circle. Use these basic constructions and the properties of geometric figures to construct other geometric figures such as perpendiculars, parallels, bisectors of angles and segments, triangles and more complex figures such as polygons. Emphasize informal deduction while doing the constructions by requiring reasons for making each construction. Do some simple locus problems by construction such as the set of points equidistant 1) from two fixed points, 2) from two fixed lines, 3) from one line, 4) from a line and a point, and 5) from a segment or a circle.

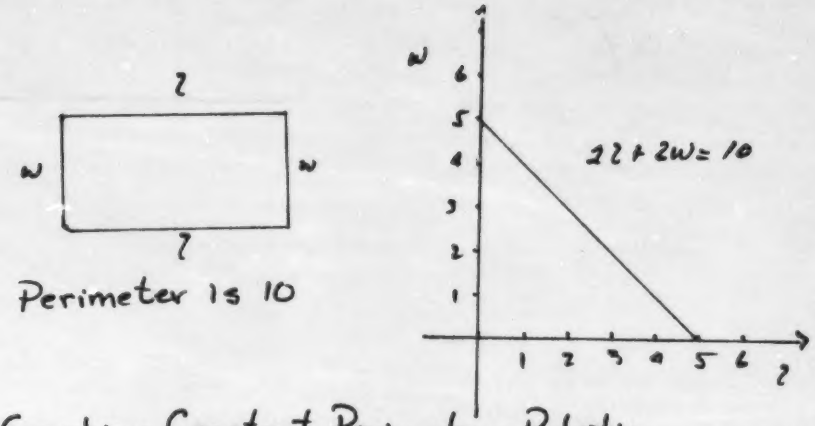
LOCUS



Locus of points equidistant from two lines

Another activity might be to find minimum conditions needed to guarantee a congruent copy of a figure such as a rectangle, a hexagon, etc. The conjectures could be checked by construction and informal reasoning.

SIMILAR FIGURES	Use the size transformation to review similar figures. Give students a figure the center and the multiplication factor and have them make similar figures. Observe what occurs when the multiplication factor is 1. Observe, in two similar shapes A and B, that the corresponding angles of A and B are congruent while the corresponding sides are related multiplicatively: side of A = k times side of B ($A=k \cdot B$). The number "k" is the
CONSTANT OF SIMILARITY	CONSTANT OF SIMILARITY. Practice finding and using "k" in different situations. Observe that if $X = k \cdot Y$ then $Y = \frac{1}{k} \cdot X$ for corresponding distances X and Y. Observe also that $X/Y = k$.
AREA	Review area and extend formulas for finding area. Introduce the formula for the area of a trapezoid. Emphasize the relationship between areas of similar shapes. Help students see that since area is calculated by multiplying two distances in the similar figures, the areas are related by a factor of "$k \cdot k$" (k^2)--one "k" for each distance. Use scale drawings to solve practical problems such as paint needed to do a room, seed needed to plant a field, etc. Re-
PERIMETER	view also the perimeter of a plane shape and a similar shape. Use the relationship to solve practical problems involving fencing and building. (Perimeter of $A = k \cdot$ perimeter of figure similar to A.)
GRAPHING	Extend GRAPHICAL work to the full four quadrant rectangular coordinate plane. Represent relations using ordered pairs of numbers; graph these relations. Given equations, produce ordered pairs, and graph them. A good source of equa-
FORMULAS	tions is the area and perimeter formulas for plane shapes--assume say a constant area, assign a value to one variable, calculate the other, and graph.



Graphing Constant Perimeter Relation for a Rectangle.

LINEAR EQUATIONS	Introduce LINEAR EQUATIONS as those whose graphs are lines. Graph lines from the equation and determine the equation given the graph. Introduce
SLOPE AND INTERCEPT	SLOPE and INTERCEPT. Graph pairs of linear equations and determine the intersection which may be a single point, empty, or the entire line. Introduce
LINEAR INEQUALITIES	the graphs of simple LINEAR INEQUALITIES.
VOLUME	The volume formulas should be extended to include those for cones, cylinders and pyramids. These formulas, along with those for the cube and box should then be used to introduce the relationship between the volumes of similar figures: Volume A = k^3·Volume B. Work on practical applications such as if Gulliver was 12 times the height of a Lilliputian, how was his weight related to that of the Lilliputian? his surface area? the cross-section of his shin bone? You might also consider the question: "Why cannot giants exist?".

STRANDS CHART

	SHAPES	SHAPE PROPERTIES	TOPOLOGICAL IDEAS	LINES, SEGMENTS
K-1	Cube, Box Cone, Cylinder Rectangle Square, Circle Triangle	Face, Edge Corner	Inside, Outside On, Path	Line Segment Number Line
2-3	Pyramid Isosceles Triangle Equilateral Triangle Parallelogram Rhombus, Kite Pentagon Hexagon Octagon	Folding Symmetry Turning Symmetry Rigidity of Figures Diagonal Diameter, Radius Cutting Symmetry for Space Shapes	Networks Skeletons	
4-5	Nonagon Decagon Polygon Right Triangle Regular Polygon Quadrilateral	Parallel Perpendicular Surface Surface Area Symmetry and Congruence Relating Shapes	Network Traversability Edges, Faces Vertices, Euler	Line Ray Number Line and Fractions Parallel, Perpendicular
6-7		Properties of Shapes Relating Properties of Shapes Informal Deduction Using symmetry	Invariants under Slides, Rotations, Reflections and Size Transformations Shadows and Rubber Sheets	Parallels and Angles
8-9	Plane Shapes in Coordinate Plane	Construction of Shapes Using Properties - Informal Deduction Locus		Equations for Lines Graphs for Lines Slope, Intercept Inequality

	MEASUREMENT	AREA	VOLUME	POSITION
K-1	Linear Concept Unit			Coordinates (Seat, Row)
2-3	Ruler Decimeter Meter Centimeter			
4-5	Perimeter Nearest Unit	Concept of Area Covering, Unit Square Unit. Area Formulas for Square, Rectangle and Right Triangle Fractions Represented as regions		
6-7		Area Formulas for Parallel-ogram, Circle and Triangle Pythagorean Relation Surface Area	Introduce Volume as Amount held by Space Shape Formulas for Cube and Box	First Quadrant Rectangular Coordinate System Polar Coordinates Bearings
8-9		Similarity and Area Formulas Trapezoid Formula	Volume of Similar Figures Volume Formulas for Cones, Cylinders and Pyramids	Full Coordinate Plane. Graphing in plane Lines, Slopes. Intercepts and Inequalities

	TESSELLATION	ANGLES	CONGRUENCE	TRANSFORMATIONS
K-1				
2-3	Tile	Corner Square, Sharp Blunt		Turn, Fold
4-5	Tessellation Slide Patterns Frieze Patterns	Obtuse, Right Acute Measuring Angles Degree Protractor	Congruent Figures Corresponding Parts Exact Copies	Measuring Turns Slide
6-7		Angles and Parallel Lines	Conditions for Congruence of Triangles-ASA SAS, SSS.	Size Transformation Reflection
8-9				Transformations in coordinate plane

	SKETCHING	SIMILARITY
K-1		
2-3		
4-5	Sketching Plane and Space Shapes	
6-7	Sketching Parallels	Concept of Similar Figures Size Transform Multiply Relations
8-9		Similar Figures and Constant of Similarity Areas, Perimeters and Volumes of Similar Figures

Appendix

Introduction to Report

The Geometry Curriculum Committee of The Michigan Council of Teachers of Mathematics was appointed in late 1977 in response to the observations and recommendations made in the NACOME Report -- "Overview and Analysis of School Mathematics Grades K-12". The Geometry Curriculum Committee was one of three curriculum study committees spawned by NACOME and the MCTM reactions to it. The other areas were consumer mathematics and fractions.

The purposes of the Geometry Curriculum Committee were to

1) provide effective arguments for including geometry in the K-12 mathematics curriculum;

2) provide guidance with regard to the content and sequence for geometry in grades K-12;

3) provide general principles for teaching geometry; and

4) provide example units, lessons, and activities for geometry at various levels of school.

The first three of these goals were accomplished for grades K-9. They are reported in these materials. It is recommended that the fourth purpose be undertaken by a group within MCTM who have interest and expertise in the area and who can use the work of this Committee as a curriculum guide.

The Board of MCTM appointed the following professionals to the Committee: Willie Aldridge, Pontiac; Connie DeMarco, Lansing; Gerri Green, Royal Oak; Esther Poage, East Lansing; Phil Cox, Waterford; Bill Fitzgerald, East Lansing; Terri Denman, Detroit; Chris Hirsch, Kalamazoo; David Reyner, Lansing; Evelyn Kozar, Detroit; Rheta Rubenstein, Detroit; and as Chairman, Art Coxford, Ann Arbor. Art Peterson, St. Johns, was later added when Ms. DeMarco was forced to resign because she left the area.

The Committee met initially in February, 1978. It met periodically for the next two years. The attached report is the result of the professional work of the total committee. We present it to you for your use with the sincere hope that its contents will help bring some order to geometry in grades K-9 and that it will be useful to school districts in Michigan and elsewhere.

For the Committee

Arthur Coxford, Ch.
May, 1980

COMMITTEE

Willie Aldridge, Pontiac
*Connie DeMarco, Lansing
Theresa Denman, Detroit
*Geraldine Green, Royal Oak
Christian Hirsch, Kalamazoo
*Esther Poage, Lansing
David Reyner, East Lansing
Philip Cox, Waterford
Evelyn Kozar, Detroit
William Fitzgerald, East Lansing
Rheta Rubenstein, Detroit
Art Peterson, St. Johns
Art Coxford, Ann Arbor, Chairman.

*Resigned before Committee completed work.

The following were members of the Guidelines Committee for Quality Mathematics Teaching during the development of this monograph:

Donald Corbin . . . . . Detroit
Arthur Coxford (Chairperson) . . . . . Ann Arbor
Sharon Hatfield . . . . . Benton Harbor
Christian Hirsch . . . . . Kalamazoo
Patricia Huellmantel . . . . . Flint
David Johnson . . . . . Ypsilanti
Virginia Keen . . . . . Quincy
Teasther Smith . . . . . Grand Rapids
Barbara Ward . . . . . Detroit

Your comments and criticism of this monograph, as well as suggestions for other monographs (or manuscripts for them) can be sent to:

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